colorado math circle

Base Phi

Euler Group

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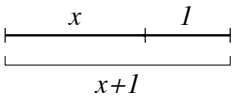

$$\frac{x+1}{x} = \frac{x}{1} \quad \Rightarrow \quad x^2 - x - 1 = 0 \quad \Rightarrow \quad x = \frac{1+\sqrt{5}}{2} = \phi$$

Base Phi (or Phinary or Phigital)

The golden ratio base is a positional number system using only the "phigits" 0 and 1. All non-negative integers can be expressed as the sum of different integer powers of ϕ. For example,

$$1 = \phi^0 = 1 \qquad 2 = \phi + \frac{1}{\phi^2} = 10.01$$

$$2 = 1 + \frac{1}{\phi} + \frac{1}{\phi^2} = 1.11$$

1. Verify that $\phi + \frac{1}{\phi^2} = 2$ and $1 + \frac{1}{\phi} + \frac{1}{\phi^2} = 2$.
2. Can you find another base phi representation for 1?
3. Find base phi representations for 3 and 4.
4. Show that 3 has an infinite number of base phi representations.
5. For any positive integer N there is a unique finite *minimal representation* that uses the fewest number of 1s. Find the minimal base phi representations (standard form) for the first ten whole numbers.
6. Show that the base phi repeating number $0.\overline{10}$ equals 1.
7. Find a base phi repeating number representation for 4.
8. How does addition work in base phi? Add 7 and 8.
9. How does multiplication work in base phi? Multiply 3 and 5.
10. Can you divide 10 by 2 in base phi?
11. Find a base phi representation for 1/2.

References

Bergman, George. "A Number System with an Irrational Base." *Mathematics Magazine* 31.2 (1957): 98–110.

Knott, Ron. "Phigits and the Base Phi representation."
`http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/phigits.html`.